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ADDENDUM

Addendum to 'Fringing fields in disc capacitors'

G J Sloggett†, N G Barton‡ and S J Spencer†

† CSIRO Division of Applied Physics, Sydney, Australia 2070
‡ CSIRO Division of Mathematics and Statistics, Sydney, Australia 2070

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Abstract. We have been made aware of further literature relating to our recent publication. In this addendum we summarise the additional material and correct an error of attribution.

We are indebted to Dr E F Kuester of the University of Colorado for drawing our attention to an error of attribution in our previous paper (Sloggett *et al* 1986) and also to some relevant literature of which we were previously unaware. Part of this literature, including Kuester's own work (1986a, b), aims at obtaining more accurate expressions for the capacitance of microstrip electrodes of various geometries, a goal similar to our own in § 4 of our paper. We summarise here the most pertinent of this additional material.

(*a*) Equation (34) is not, as stated, the first-order approximation to the capacitance of a disc and plane proposed by Ignatowsky. It was based on an erroneous account of this formula given by Sneddon (1966). The correct Ignatowsky formula is

$$C \approx C_{\mathrm{elem}}\left(1+\frac{2d}{\pi R}\ln\frac{4R}{d\sqrt{e}}\right).$$

The effect of this change is to increase significantly the errors of this approximation, displacing curve C in figure 5 to a position well below that of any other curve. Another historically important first-order approximation is that of Clausius (1852):

$$C \approx C_{\mathrm{elem}}\left(1+\frac{2d}{\pi R}\ln\frac{4R}{\alpha^2 e^2 d}\right).$$

If the value $\alpha = 0.247$ determined by Clausius is substituted, an approximation which is between the Kirchhoff-Hutson and Maxwell-Cooke formulae is obtained.

(*b*) The higher-order capacitance expression, equation (36), was obtained by deleting from an expression of Shaw (1970) a term having the coefficient $C^{(3)}$. Wigglesworth (1972) has shown that $C^{(3)}$ is equal to zero, implying that equation (36) is in fact equivalent to Shaw's expression. A quite different higher-order approximation for the capacitance of a disc and plane is given by von Guttenberg (1953) (equation (3), p 338). We have checked it against the numerical data used in compiling figure 5, and have found it to have rather large positive errors, e.g. +25.3% at $R/d = 2$, and increasing for decreasing R/d.

(*c*) A number of authors have given expressions for the capacitance of parallel-plate systems which are more general than those discussed here in that they apply to electrodes of arbitrary shape and to the case where the interelectrode medium has a relative permittivity differing from that of the environs (i.e. the microstrip case). Two such expressions, building on earlier work, are given by Kuester (1986a), one applying to electrodes whose dimensions are large compared with the electrode separation and the other to small electrodes. The large electrode approximation generalises the Kirchhoff–Hutson formula, to which it has similar accuracy. Like most of the more general formulae, it cannot be evaluated explicitly in the general case, although the problem is shown to be intimately connected with that of determining the inductance of a loop of arbitrary shape. Inductances, and hence capacitances, for various shapes are given in the literature, and others may be computed by evaluating a double integral.

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